**ENGN2020 – HOMEWORK3**

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### Problem 2(K8-4-5)

Matrix A =, Matrix P =

Then, P-1 = , therefore

For matrix A, the characteristic determinant gives the characteristic equation:

Then we get:

The roots, which are also eigenvalues of A, are , ,

For

= =

Let , then ,

So, an eigenvector of A corresponding to is

For

==

Let , then ,

So, an eigenvector of A corresponding to is

For

=

, and , can be anything, let

So, an eigenvector of A corresponding to is

For matrix , the characteristic determinant gives the characteristic equation:

The roots, which are the same eigenvalues of A, are , ,

For

Let , then ,

So, an eigenvector of corresponding to is

For

Let , then ,

So, an eigenvector of corresponding to is

For

, and , can be anything, let

So, an eigenvector of corresponding to is

It can be calculated that

### Problem 4(K20-7-1 to K20-7-6)

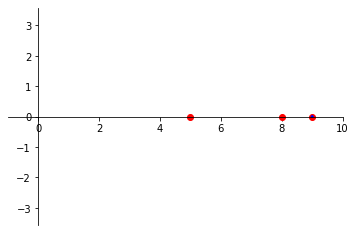
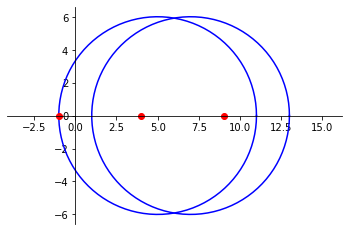
******(a) K20-7-1:**  **(b) K20-7-2**

Figure 1: Gerschgorin disks of K20-7-1 Figure 1: Gerschgorin disks of K20-7-2

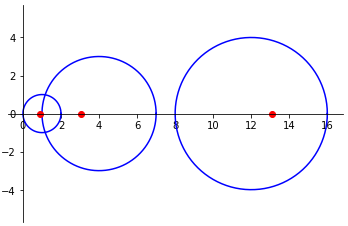
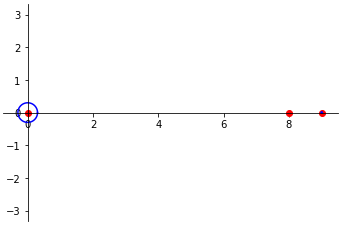
******(c) K20-7-3:** **(d) K20-7-4:**

Figure 3: Gerschgorin disks of K20-7-3 Figure 4: Gerschgorin disks of K20-7-4

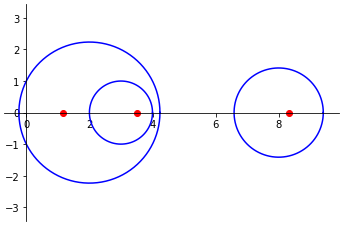
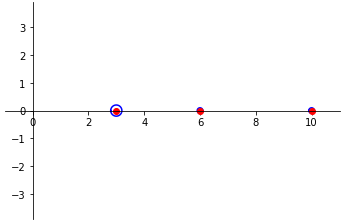
**(e) K20-7-5:**  **(f) K20-7-6:**

Figure 5: Gerschgorin disks of K20-7-5 Figure 6: Gerschgorin disks of K20-7-6

### Problem 5(K8-4-6)

**(a)Trace:**

Let A be a matrix, .

For , trace A = 0, its eigen values are 5 and -5, the trace equals the sum of the eigenvalues.

For , trace A = 10, its eigen values are 6 and 4, the trace equals the sum of the eigenvalues.

For , trace A = 14, its eigen values are 0, 4 and 10, the trace equals the sum of the eigenvalues.

**(b)Trace of product:**

Proof:

Let A be a matrix, .

Let B be a matrix, .

, where , then

, where ,then

Therefore,

**(c)** **Relationship between and :**

Let A and P be matrices

Let and

**(d)** **Diagonalization:**

To change the order of the eigenvalues in D, we should interchange the corresponding eigenvectors in matrix X in

### Problem 6(K8-5-1 to K8-5-6)

**(a) K8-5-1:**

, then , , so A is a Hermitian matrix.

According to Theorem 1, the eigenvalues are real numbers.

In fact, the eigenvalues are -6.08276253 and 6.08276253

**(b) K8-5-2:**

, then , , so A is a skew-Hermitian matrix.

According to Theorem 1, the eigenvalues are pure imaginary numbers or zero.

In fact, the eigenvalues are and

**(c) K8-5-3:**

, then , , so A is a unitary matrix.

According to Theorem 1, the eigenvalues have absolute value 1

In fact, the eigenvalues are and

**(d) K8-5-4:**

, then , , so A is a skew-Hermitian matrix.

According to Theorem 1, the eigenvalues are pure imaginary numbers or zero.

In fact, the eigenvalues are and

**(e) K8-5-5:**

, then , , so A is a skew-Hermitian matrix.

According to Theorem 1, the eigenvalues are pure imaginary numbers or zero.

In fact, the eigenvalues are , and

**(f) K8-5-6:**

then , , so A is a Hermitian matrix.

According to Theorem 1, the eigenvalues are real numbers.

In fact, the eigenvalues are 4, 0 and -4.

### Problem 7

**(a) Fixed-point iteration:**

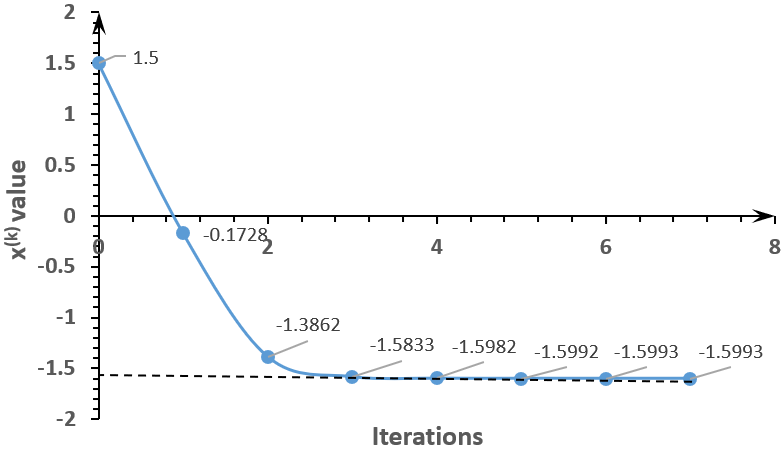


Figure 7: Fixed point iteration

**(b)Newton-Raphson iteration:**

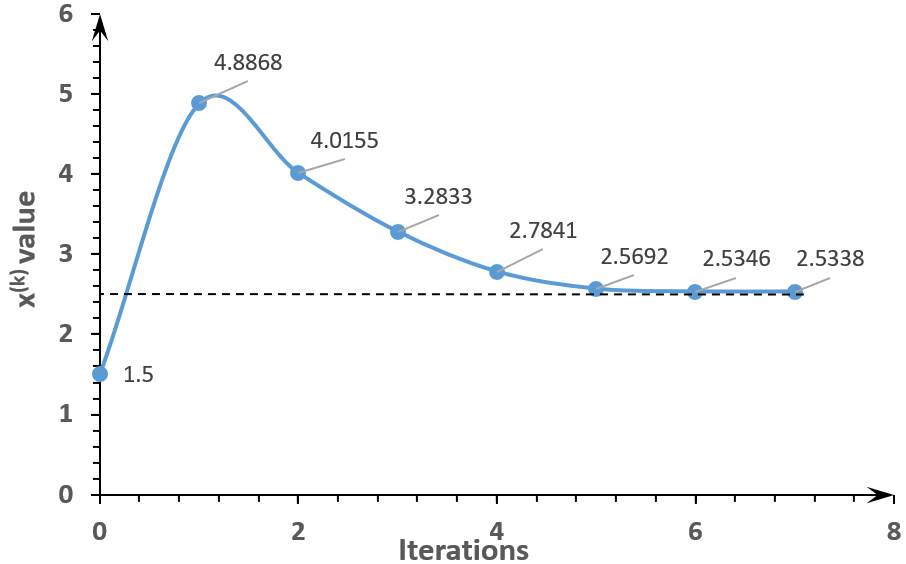


Figure 8: Newton-Raphson iteration

### Problem 8

**(a) Plots and roots**

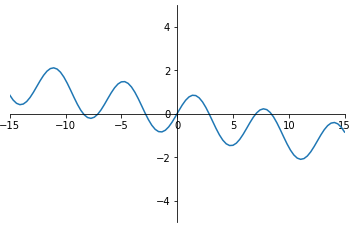


Figure 9: Plot of

There are seven roots. The roots are -8.4232, -7.0681, - 2.8523, 0, 2.8523, 7.0681, 8.4232.

**(b) Factor out one root**

Choose 8.4232 as the root to be factored out.

Then

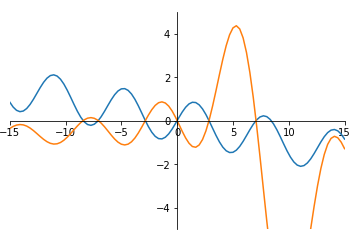


Figure 10: Plot of and . Blue line for , and yellow line for

For , there are six roots. The roots are -8.4232, -7.0681, - 2.8523, 0, 2.8523, and 7.0681. They are the same as the roots of , except for

**(c) Factor out three roots**

Choose 8.4232, -7.0681 and 2.8523 as the roots to be factored out.

Then

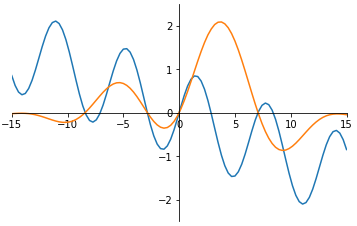


Figure 11: Plot of and . Blue line for , and yellow line for

For , there are four roots. The roots are -8.4232, - 2.8523, 0, and 7.0681. They are the same as the roots of , except for 8.4232, -7.0681 and 2.8523.

**(d) Problems**

1.For some equations, x = 0 can be a possible root. However, it`s unable with that root.

2.By factoring out some roots, the function g(x) may become quite complicated, and it`s even harder to get the derivative.